Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐☐   Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (2011 Onwards)   (Sem.–2)
ENGINEERING MATHEMATICS – II
Subject Code : BTAM-102
Paper ID : [A1111]

Time : 3 Hrs.   Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :
1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION A

1. **Write briefly :**

   a) Define Linear independent and dependent vectors.

   b) Find the characteristic equation of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

   c) Test whether the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is diagonalisable or not?

   d) Express $\sin^5 \theta$ in terms of the sines of multiples of θ.

   e) Find the general and principle value of i^i.

   f) Examine the convergence / divergence of the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{(1+\frac{1}{n})^{n^2}}$.

   g) Define the term absolute convergence and use this concept to test the convergence of the series $\sum\limits_{n=1}^{\infty} \dfrac{\sin n}{n^2}$.

h) Find the degree and order of the differential equation $\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \dfrac{d^2y}{dx^2}$

i) For what value of k the equation $xy^3 dx + k(x^2 y^2)dy = 0$ is a exact equation.

j) Find the complete solution of the equation :

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0, \text{ given } x(0) = 0, \frac{dx}{dt}(\text{at } t = 0) = 15.$$

SECTION-B

2. a) Use method of variation of parameters to find the general solution of the differential equation $y'' + y = \operatorname{cosec} x$.

   b) Find the complete solution of the differential $y'' - 2y' + y = x \sin x$.

3. a) Solve the following simultaneous differential equation

   $$\frac{dx}{dt} + y + 5x = e^t, \frac{dy}{dt} + x + 5y = e^{5t}$$

   b) Find the complete solution of the differential equation $x^2 y'' - 3xy' + 5y = x \log x$ by using operator method.

4. a) Solve the differential equation : $\dfrac{dy}{dx} = \dfrac{y+1}{(y+2)e^x - x}$.

   b) Find the particular solution of the differential equation $y'' - 2y' + 2y = e^x \tan x$

5. An electric circuit consists of an inductance 0.1 henry, a resistance of 20 ohms, and a condenser of capacitance 25 microfarads. Find the charge q and the current i at time t, given the initial conditions $q = 0.05$ coulombs, $i = 0$ when $t = 0$.

SECTION-C

6.   a) Use the rank method to test the consistency of the system of equations

$x + 2y + z = 2$; $3x + y - 2z = 1$; $4x - 3y - z = 3$; $2x + 4y + 2z = 4$, if consistent, then solve it completely.

      b) Use Gauss-Jordan method to find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 5 \end{pmatrix}$$

7.   a) Verify Cayley-Hamilton theorem for the matrix

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

        and hence find its inverse.

      b) Prove that eigen values of Hermetian matrix are real.

8.   a) Test the convergence of the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \ldots\ldots\ldots\infty (x > 0)$$

      b) Test for what values of x does the series convergence/diverge?

$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}} x^n$$

9.   a) Use Demoivre's theorem to prove that

$$\left(\frac{1 + \sin\alpha + i\cos\alpha}{1 + \sin\alpha - i\cos\alpha} \right)^n = \cos\left(\frac{n\pi}{2} - n\alpha \right) + i\sin\left(\frac{n\pi}{2} - n\alpha \right)$$

      b) Separate $\tan^{-1}(e^{i\theta})$ into real and imaginary parts.